

Structured experimental modeling of complex nonlinear systems

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Outline

- Introduction
- Nonlinear SM identification
- Structured experimental modeling
- Application to identification of vehicles vertical dynamics
- Conclusions

Introduction

- The problem of **using data and physical information** in the identification of complex nonlinear systems is considered.
- When possible, physical laws of involved phenomena are used to derive the structure of the model, depending on some parameters, whose values are tuned using measured data.
- In many practical applications the required physical laws are too complex to derive accurate structures.



large identification errors

- In such cases input-output (black-box) models are used.



physical information difficult to account for

Introduction

- *Experimental modeling*: input-output or black-box models are used, e.g. in regression form:

$$w_t^i = [y_t^i \dots y_{t-n_y+1}^i \quad u_t^1 \dots u_{t-n_1+1}^1 \dots u_t^m \dots u_{t-n_m+1}^m] \quad y_{t+1}^i = f_o^i(w_t^k), \quad i = 1, \dots, q$$

Problem:

*Find estimate \hat{f} of $f_o = [f_o^1, \dots, f_o^q]$
possibly giving “small” identification error*

- The usual approach is to assume that f_o belongs to a finitely parametrized set of functions $f(\theta)$. Data are used to estimate the parameters θ .
- Basic to this approach is the choice of the set of functions $f(\theta)$, typically realized by some search on different functional forms: linear, bilinear, polynomial, neural networks, etc.

Drawbacks: — this search may be quite time consuming and
— leads to approximate model structures

Nonlinear SM identification

- References:

- *M. Milanese and C. Novara, “Optimality in SM Identification of Nonlinear Systems”, SYSID 2003, Rotterdam, the Netherlands, 2003.*
- *M. Milanese and C. Novara, “Set Membership estimation of nonlinear regressions”, IFAC 2002, Barcelona, Spain.*

- Key features:

- The method allows to identify nonlinear model sets.
- No assumptions on the functional form of f_o are required. Regularity assumptions, given by bounds on the gradient of f_o , are used.
- No statistical assumptions on noise are made. Noise is only supposed to be bounded.
- The complexity/accuracy problems posed by the choice of the suitable parametrization of the nonlinear regression function are circumvented.

**Alternative approach:
Structured (block-oriented) modeling**

*use of information on
the physical interconnection structure*



*decomposition in
interacting subsystems*



*identification of low dimensional subsystems
and estimation of their interactions*

Structured experimental modeling

- Typical cases considered in the literature: Hammerstein, Wiener and Lur'e systems.

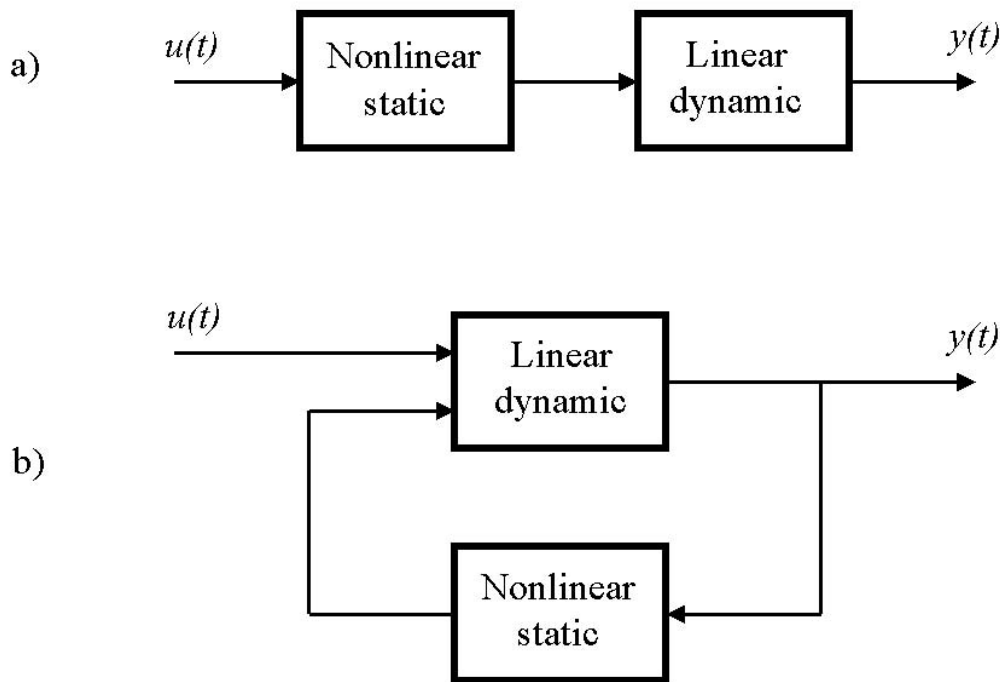


Figure 1: a) Hammerstein System. b) Lur'e system.

- In practical applications more complex structures may be needed, e.g. composed of many subsystems and with nonlinear dynamic blocks.

Structured experimental modeling

The following decomposition structure is considered:

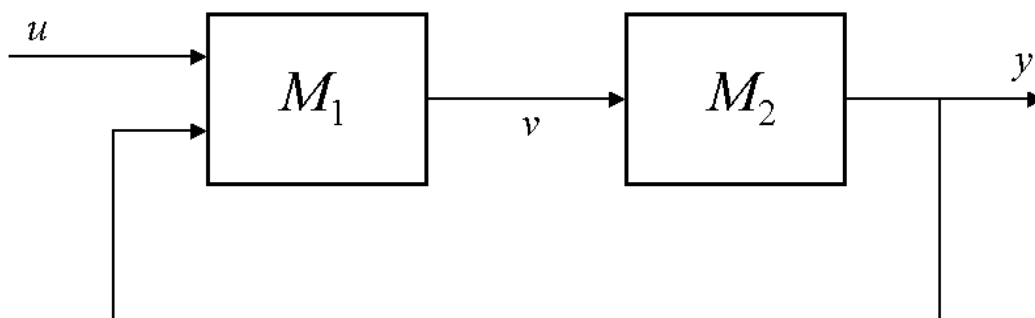


Figure 2: Structure decomposition.

- All the signals u, y, v may be multivariable.
- Submodels M_1 and M_2 are dynamic MIMO discrete time systems. M_1 is nonlinear and M_2 is linear.
- $v = M_1[u, y]$, $y = M_2[v] = M[u]$

Problem:

Identify M_1 and M_2 , supposing that u, y are known, but v is unknown.

Note: Hammerstein, Wiener and Lur'e models are particular cases.

Structured identification algorithm

- Initialization:
 - Partition the data in estimation data set u^e, y^e and validation data set u^v, y^v
 - Get an initial guess $M_2^{(o)}$ for M_2 and set $M_1^{(o)} = 0$
 - Set $k=1$
- Iteration k :
 1. Compute a sequence $v^{(k)}$ such that $M_2^{(k-1)}[v^{(k)}] \approx y^e$
 2. Identify a nonlinear regression model $\bar{M}_1^{(k)}$ using u^e and y^e as input sequences and $v^{(k)}$ as output sequence
 3. Identify a linear model $\bar{M}_2^{(k)}$ using $\bar{v}^{(k)} = \bar{M}_1^{(k)}[u^e, y^e]$ as input sequence and y^e as output sequence
 4. Compute $\alpha^* = \arg \min_{\alpha \in R^2} J(\alpha, k)$
 where:

$$J(\alpha, k) = \|y^v - y_\alpha^{(k)}\|_2^2$$

$$y_\alpha^{(k)} = M(M_1^\alpha, M_2^\alpha)[u^v]$$

$$M_1^\alpha = M_1^{(k-1)} + \alpha_1(\bar{M}_1^{(k)} - M_1^{(k-1)})$$

$$M_2^\alpha = M_2^{(k-1)} + \alpha_2(\bar{M}_2^{(k)} - M_2^{(k-1)})$$
 5. Set $M_1^{(k)} = M_1^{\alpha^*}$, $M_2^{(k)} = M_2^{\alpha^*}$, $k = k + 1$ and return to step 1

Structured experimental modeling

- Let $M_*^{(k)}$ be the overall model identified from the estimation data set (u^e, y^e) at iteration k
- The quality of $M_*^{(k)}$ is measured by the **simulation error on the validation data set** (u^v, y^v) :

$$J^*(k) = \|y^v - M_*^{(k)}[u^v]\|_2^2$$

- Choosing $\alpha = 1$, the algorithm reduces to classical iterative algorithm proposed for Hammerstein model.
- With $\alpha = 1$, $J^*(k)$ may blow up for increasing k . This cannot happen using the proposed algorithm, as stated in the following proposition.

Proposition

$$J^*(k+1) \leq J^*(k), \quad \forall k$$



Structured experimental modeling

- In step 1, $v^{(k)}$ can be computed as $v^{(k)} = M_2^\dagger[y^e]$ where M_2^\dagger is an approximate stable inverse of $M_2^{(k)}$ computed by solving the following H_∞ optimization problem:

$$M_2^\dagger = \arg \min_{Q \in \mathcal{H}_\infty} \|[1 - M_2^{(k)}Q]W\|_\infty$$

W is a low pass filter chosen on the basis of the spectral features of measured signals y and of noise affecting such measurements.

- Even in case a stable right inverse of $M_2^{(k)}$ exists, the use of M_2^\dagger instead of the exact inverse is preferable, in order to avoid unduly amplification of the effects of noise outside the frequency range of interest for y .
- In step 2, the Nonlinear SM identification method can be used in order to avoid the need of extensive and time consuming searches of suitable functional forms of the regression function.

Identification of vehicles with controlled suspensions

- Identification is performed on simulated data obtained by the following half-car model:

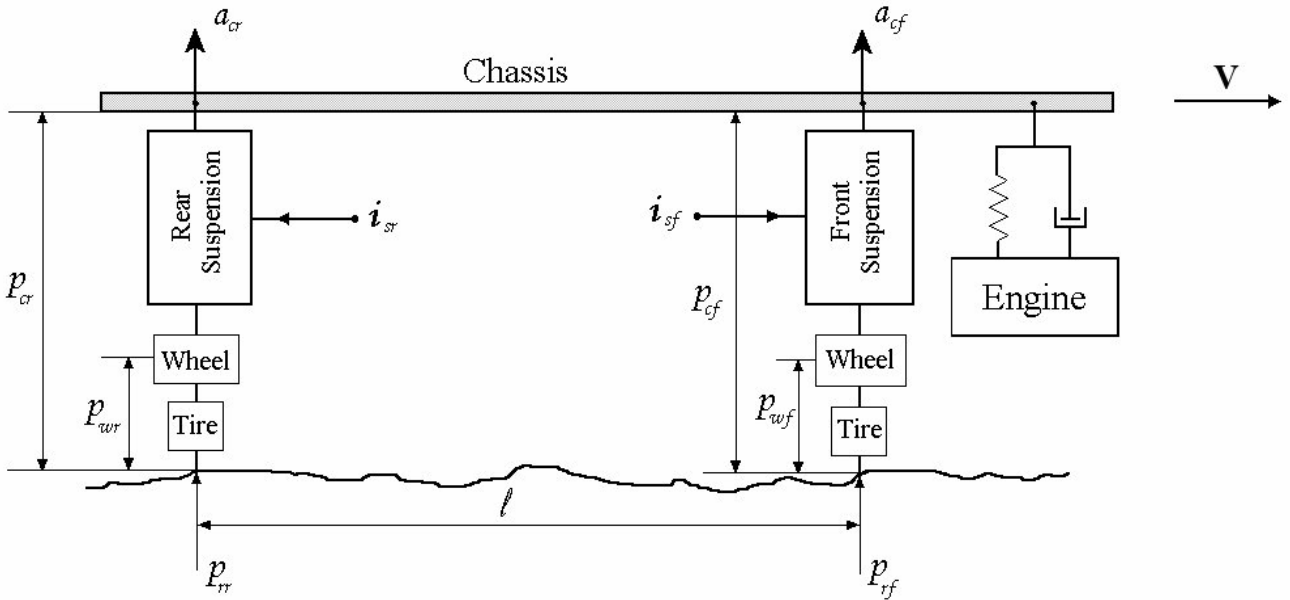


Figure 3: The half-car model.

- p_{rf} and p_{rr} : front and rear road profiles.
- i_{sf} and i_{sr} : control currents of front and rear suspensions.
- a_{cf} and a_{cr} : front and rear chassis vertical accelerations.
- p_{cf} and p_{cr} : front and rear chassis vertical positions.
- p_{wf} and p_{wr} : front and rear wheels vertical positions.
- F_{cf} and F_{cr} : forces applied to chassis by front and rear suspensions.
- F_{wf} and F_{wr} : forces applied to front and rear wheels by tires.

Identification of vehicles with controlled suspensions

- The chassis, the engine and the wheels are simulated as rigid bodies.
- Static nonlinearities have been considered for suspensions and tyres:

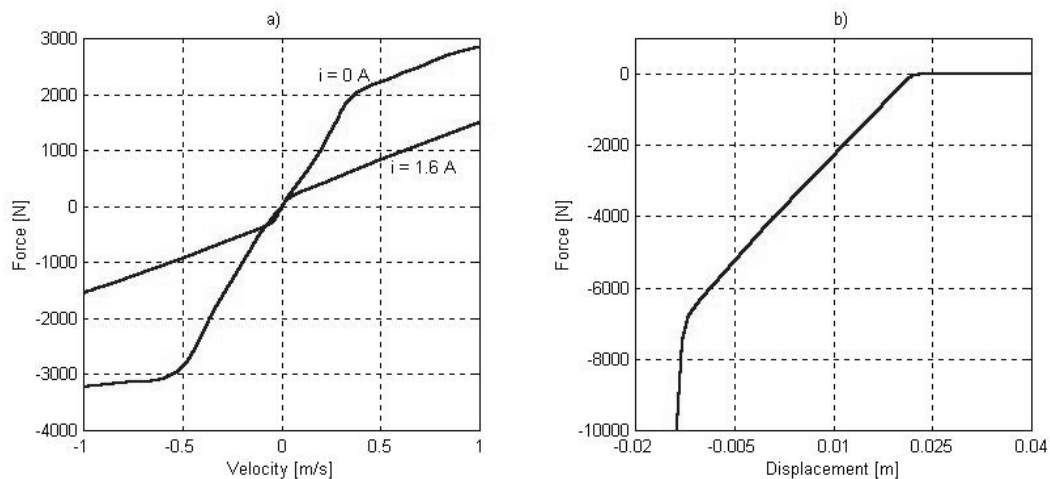


Figure 4: a) Force-velocity characteristic of suspension. b) Force-displacement characteristic of tires.

The data

- Data set generated from “true system” simulation:
 - Recorded variables: p_{rf} , p_{rr} , i_{sf} , i_{sr} , a_{cf} and a_{cr} .
 - Sampling time: $\tau = 1/512$ sec.
 - Road profile: random with amplitude ≤ 2.6 cm.
 - Estimation set:
 - 10240 data corresponding to 20 seconds of “true system” simulation.
 - Corrupted by a uniformly distributed noise of relative amplitude 5%.
 - Used for models identification.
 - Validation set:
 - 2049 data, corresponding to 4 seconds of “true” data not used for estimation.
 - Used to test the simulation accuracy of identified models.
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Unstructured identification of vehicles vertical dynamics

- Measured variables: p_{rf} , p_{rr} , i_{sf} , i_{sr} , a_{cf} and a_{cr} .
- Unstructured identification: Two nonlinear model, not using information on system structure, have been identified:
 - NSMU: Nonlinear Set Membership
 - NNU: Neural Network

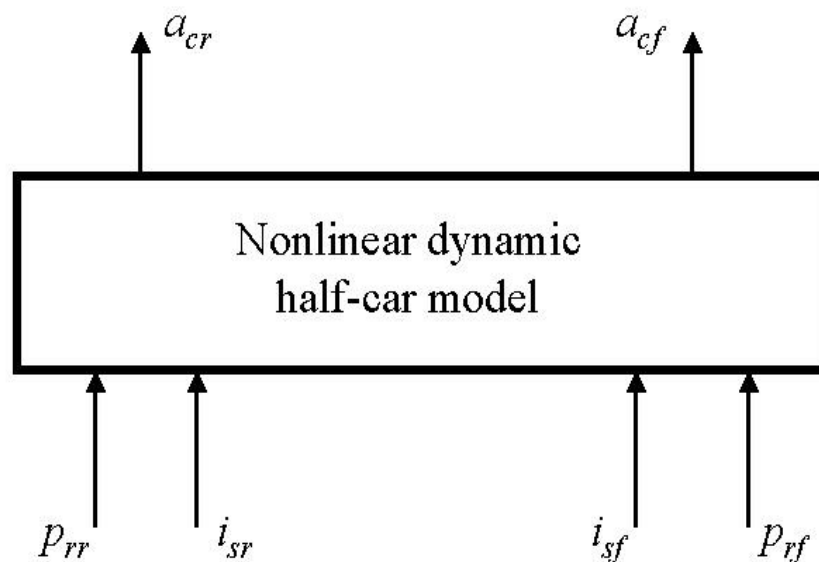


Figure 5: Unstructured model blocks diagram.

Structured identification of vehicles vertical dynamics

- Measured variables: the same as for unstr. identification.
- Structured identification: The system is decomposed in sub-systems:

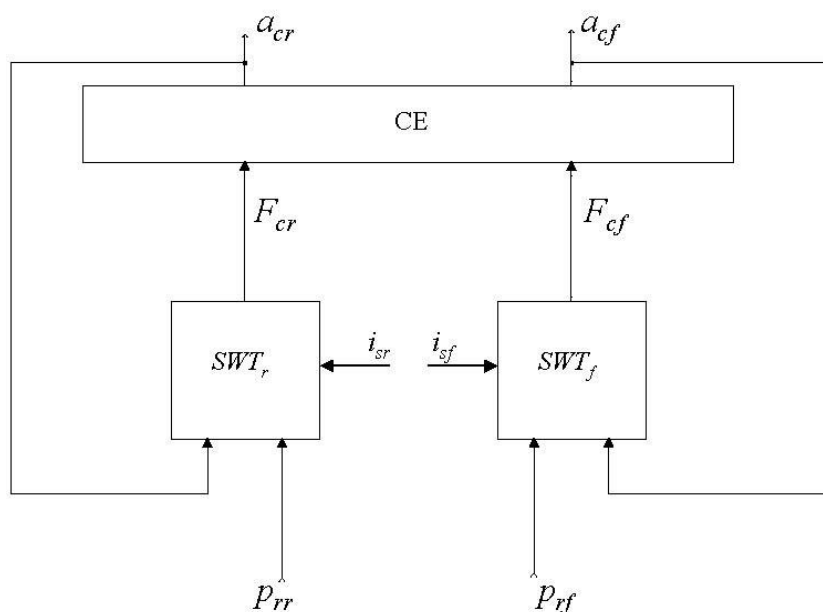


Figure 6: Generalized Lur'e form of half-car model.

- CE : Chassis + engine
- SWT : Suspension + wheel + tyre

Note: The forces F_{cf} and F_{cr} are not measured.

Structured identification of vehicles vertical dynamics

- A model, called NSMS, has been identified using the structured identification algorithm, with the following choices:
 - The initial model required at step 1 has been obtained from the laws of motion, assuming chassis and engine as a unique rigid body.
 - In step 3, discrete time nonlinear models of SWT_f and SWT_r were identified using the Nonlinear SM approach.
 - In step 4, the models of CE are linear Output Error models identified by means of the Matlab Identification Toolbox, with inputs F_{cf} , F_{cr} and outputs a_{cf}, a_{cr} .
- Significant improvements of chassis accelerations errors have been obtained after 2 iterations.
- A third iteration has been also performed, but no significant decrease of errors have been observed.

Simulation results

| Model | NSMU | NNU | NSMS ⁽¹⁾ | NSMS ⁽²⁾ |
|-------------|-------|-------|---------------------|---------------------|
| <i>RMSE</i> | 0.656 | 0.660 | 0.502 | 0.208 |

Table 1: Root mean square front chassis acceleration errors on the validation data set.

Structured identification of vehicles vertical dynamics

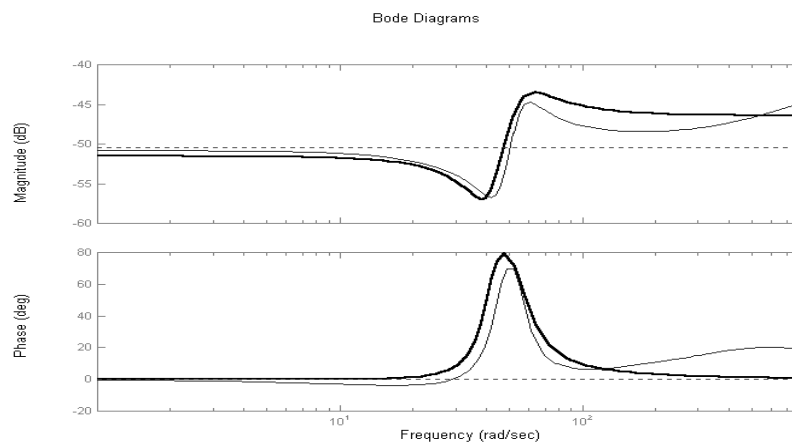


Figure 7: Bode plots of $CE(1,1)$: “true” (bold line), initial model (dashed line), model at iteration 2 (thin line).

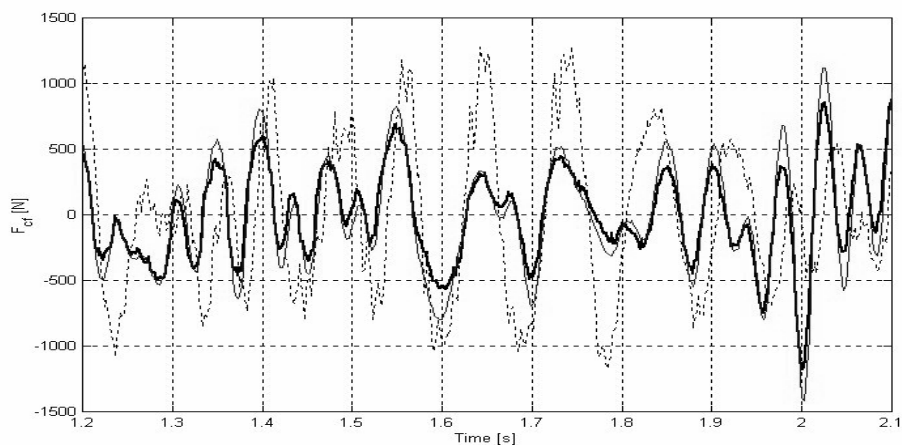


Figure 8: Forces applied to chassis from suspensions: “true” (bold line), estimate at iteration 1 (dashed line), estimate at iteration 2 (thin line).

Simulation results

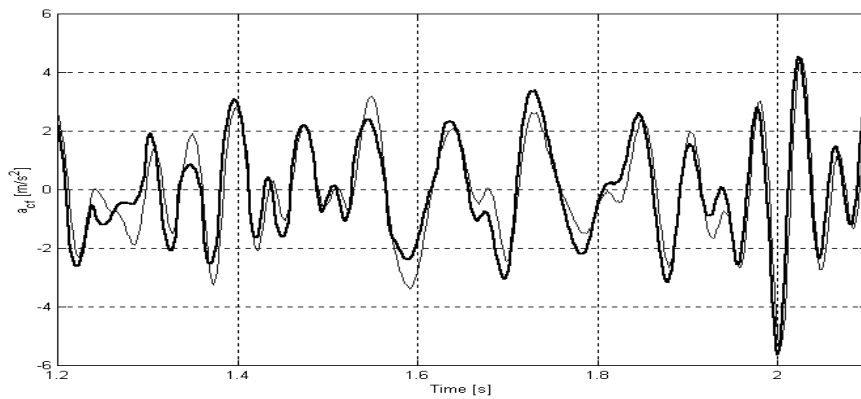


Figure 9: Front chassis accelerations: “true” (bold line), NSMU model (thin line).

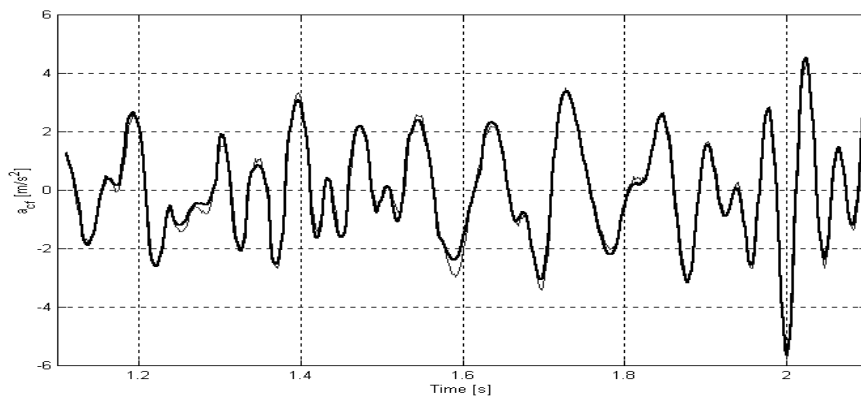


Figure 10: Front chassis accelerations: “true” (bold line), NSMS⁽²⁾ model (thin line).

Conclusions

- An approach for block-oriented identification able to deal with complex structures has been proposed. The key features of the approach are:
 - The nonlinear subsystems can be dynamic.
 - The nonlinear subsystems are not supposed to have a given parametric form.
 - A structured identification algorithm has been presented, guaranteeing that the identification error does not increase at each algorithm iteration.
 - The effectiveness of the approach has been tested on a simulated half-car model for vehicles vertical dynamics.
 - The identified structured models in few iterations reduced significantly the simulation errors, largely improving over unstructured models and reaching quite satisfactory identification accuracy.
 - The algorithm has also been used with success in identification from experimental data acquired on a real car.
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